New Superembeddings for Type II Superstrings D. V. Uvarov

Possible ways of generalization of the superembedding approach for the supersurfaces with the number of Grassmann directions being less than the half of that for the target superspace are considered on example of Type II superstrings. Focus is on $n=(1,1)$ superworldsheet embedded into $D=10$ Type II superspace that is of the interest for establishing a relation with the NSR string. Superstrings and Heterotic Strings, Supersymmetry Breaking, Supergravity Models document equation0

Introduction The interpretation of superbranes as solitons of supergravity/super-Yang-Mills theories for the first time suggested by J. Hughes et.al. HughesLP (for reviews see Soliton) allows to connect partial breaking of target space supersymmetries caused by these solitonic configurations with $\kappa-$symmetry inhereit to the brane worldvolume actions. In particular, for conventional superbranes that leave unbroken half of the target space supersymmetries $\kappa-$symmetry has the same number of independent parameters. In the proper gauge the action of $\kappa-$symmetry transformations for one half of the target space Grassmann coordinates η coincides with that of supersymmetry transformations. It is these coordinates that are associated with the unbroken part of target space supersymmetries and they can be identified with Grassmann coordinates on the superworldvolume. The other components of the target space Grassmann coordinates θ' constitute Goldstone fermions associated with the broken supersymmetries.

The worldvolume nature of $\kappa-$symmetry was revealed later by D.V. Volkov and collaborators VZ-STVZ. (It should be noted, that the early attempts to combine the worldsheet and target-space supersymmetries for particles and strings were made in the middle of 80's Gates. However, suggested method was too general to describe conventional models.) This inspired the investigation of worldvolume superfield formulations of superbranes DPSF-Drum, where $\kappa-$symmetry is replaced by the more fundamental worldvolume superdiffeomorphism symmetry. These worldvolume superfield actions upon integration over the worldvolume Grassmann variables and the elimination of auxiliary fields by means of the equations of motion have to reduce to the Green-Schwarz-type actions, where $\kappa-$symmetry appears as a remnant of the local worldvolume superdiffeomorphism symmetry. Further gauge fixing of the local symmetries for these Green-Schwarz-type actions yields the theory with equal numbers of the worldvolume bosons and fermions. So the role of $\kappa-$symmetry is to assemble local symmetries gauge fixed bosonic and fermionic degrees of freedom of the brane into the worldvolume supermultiplets. This is well known bose-fermi matching. Taken as the key principle for constructing superbranes it allows to define possible values for p and D (dimension of the target superspace), or, the brane scan. However, it turns out that for some types of branes like Type II superstrings in $D=4,6,10$ Galperin, BPSTV, $D=11$ supermembrane BPSTV, $M5-$brane BLNPST the construction of the worldvolume superfield actions with $\kappa-$symmetry being entirely replaced by the worldvolume superdiffeomorphisms encounters significant difficulties just in the point of the relation to the corresponding Green-Schwarz-type actions since the present in the doubly supersymmetric formulations auxiliary fields become dynamical.

The systematical study of doubly supersymmetric brane formulations is achieved in the framework of the superembedding approach (for review see DS). This is the supersymmetric generalization of the theory of surface embeddings Eis and deals with a brane superworldvolume embedded into a target superspace. Initially the idea of the consideration of bosonic strings as surfaces embedded into the target space was put forward in RLO (see also BN). The relation with the gauge theories on the worldvolume was studied in Zh81. The first indications of the efficiency of embedding approach in the supersymmetric theories of particles and strings were discovered in VZ-STVZ. There was also introduced the concept of geometrodynamical equation as the basic ingredient of superembedding approach. Detailed analysis of the case when the number of the superworldvolume Grassmann directions n_{wv} equals half the number of the target superspace Grassmann coordinates n_{ts} was performed in BPSTV.

In supersymmetric theory of embeddings there appears the interesting phenomenon that has no analogs in the classical bosonic theory, namely, for certain branes geometrodynamical equation contains superbrane equations of motion. This leads to the new class of on-shell embeddings. It is precisely for the branes of this class one faces the described above obstacles when trying to construct worldvolume superfield models. (It is possible, however, to construct worldvolume superfield actions for the gauge fixed physical degrees of freedom in the framework of the method of nonlinear realizations. This technique is also applicable for the off-shell superbranes (see Ivanov and references therein).) On the other hand, for on-shell embeddings the basic superembedding equation encompasses full information about the superbrane dynamics. If the superembedding is off-shell then the superbrane equations of motion should be imposed additionally. The situation is similar to that in the bosonic theory Eis-Zh81. Then one is able to construct the doubly supersymmetric superfield actions for corresponding superbranes. This scheme was realized for various superparticle models STV-BMS, null-superstrings BSTV, heterotic superstrings Berkovits1-BB, $D=3$ Type II superstring Galperin (see also CP), $D=4$ $N=1$ supermembrane HRS, PST and the space-filling branes in $D=3,4$ BPPST, Drum.

The characteristic feature of the superembeddings considered so far is that the number of the superworldvolume Grassmann directions n_{wv} equals to the half of the number of target superspace Grassmann directions n_{ts}. This situation corresponds to the most complete description of superbranes, where $\kappa-$symmetry of component formulations is entirely replaced by the local worldvolume supersymmetry. However, it is possible to contemplate more general situation with $n_{wv} < n_{ts}/2$ (the case $n_{wv} > n_{ts}/2$ seems to be too restrictive although there exist superparticle models BLS with more than $n_{ts}/2$ $\kappa-$symmetries) in an attempt to extend the class of off-shell superembeddings for which exist worldvolume superfield formulations. Considered in the literature worldvolume superfield actions for particles and strings with $n_{wv} < n_{ts}/2$ VZ-STVZ, Berkovits1, Tonin, Berkovits2, Berkovits3, DIS, CP, APT, for which $\kappa-$symmetries that were not realized as worldvolume superdiffeomorphisms appear as the local symmetries on the superfield level, suggest that such embeddings can still describe breaking of the half of target space supersymmetries, however, there can also exist configurations that break larger fraction of supersymmetry. It therefore seems to be instructive to study such embeddings with $n_{wv} < n_{ts}/2$ in the framework of the superembedding approach. In the present paper we consider $D=10$ Type II superstrings as the representatives of on-shell (when $n_{wv} = n_{ts}/2$) superembeddings. We concentrate on $n=(1,1)$ superworldsheet embedded into the flat $D=10$ Type II target superspace that is of the most interest for establishing the relation with the NSR string VZ-STVZ, Berk-me invariant under the local $n=(1,1)$ worldsheet supersymmetry.

Basic equations

The necessary ingredients of our construction are vector and spinor Lorentz harmonic variables GIKOS-harmonics $(u_{\underline{m}}^{(n)}, v_{\underline{\mu}}^{(\alpha)})$ and their inverse $(u^{\underline{m}}_{(n)}, v^{\underline{\mu}}_{(\alpha)})$. For the $D=10$ target superspace these are 10×10 and 16×16 orthonormal matrices respectively. Spinor harmonics also should satisfy certain harmonicity conditions that reduce the number of independent degrees of freedom of the spinor harmonics to the dimension of $SO(1,9)$ Lorentz group equal to 45. In the presence of the superstring $SO(1,9)_R$ group acting on the index in brackets of harmonics is broken down to $SO(1,1)\times SO(8)$, thus the Lorentz harmonics adapted for the description of superstrings acquire the form equation1 $\mathrm{u}_{\underline{m}}^{(n)} = \left(u_{\underline{m}}^{+2}, u_{\underline{m}}^{i}, u_{\underline{m}}^{-2}\right), v_{\underline{\mu}}^{(\alpha)} = \left(v_{\underline{\mu}q}^{+}, v_{\underline{\mu}\dot{q}}^{-}\right), v^{\underline{\mu}}_{(\alpha)} \equiv \left(v_{\underline{\mu}}^{(\alpha)}\right)^{-1} = \left(v_q^{-\underline{\mu}}, v_{\dot{q}}^{+\underline{\mu}}\right)$